Chapter 8 Project: Lead in the Body

Name_____________________________ Name ______________________________

Introduction:

Lead is a heavy metal that can damage the neurological system and kidneys in humans. Although lead is not used anymore in most paints and gasoline products, it is still present in many drinking water systems. The source of lead in drinking water is typically the solder holding the pipes together, not the pipes themselves. Water quality tests conducted in 2004 in the Seattle Public School system revealed that several elementary schools had high lead levels in water from drinking fountains.[1] Many fountains that were tested had lead concentrations over 100 parts per billion (ppb), with lows of around 5 ppb to a high value of over 1,600 ppb. The U.S. Environmental Protection Agency (EPA) standard for lead in drinking water is 15 ppb.[2]

In this project you will mathematically model the blood lead levels in Katrina, a 10-year-old who enrolls in a school that has lead in the drinking water. How does the amount of lead in Katrina's bloodstream increase? How long does it take for Katrina to become "lead poisoned"?

1. Daily intake of lead

Suppose Katrina enrolls at a school and is placed into a classroom near a drinking fountain where the lead concentration in the water is at the EPA standard of 15 parts per billion (ppb) *by mass*. This means that there are 15 grams of lead for every billion grams of water:

$$\text{lead concentration} = \frac{15 \text{ g lead}}{1{,}000{,}000{,}000 \text{ g } H_2O}$$

a) Assume that each day Katrina drinks about ¾ of a liter of water from the drinking fountain near her classroom. Determine the number of grams of lead that Katrina consumes each day from the school's water. *Recall that 1 liter of water has a mass of 1,000 grams.*

b) Micrograms are a more common unit of measure when working with small masses such as the amount of lead in the body. Convert your last answer into micrograms (μg). *Recall that there are a million micrograms in a gram. Show unit conversion.*

c) Not all of the lead that people consume is absorbed into the bloodstream. Adults absorb less than 5% of consumed lead, whereas children can absorb up to 50%.[3] This is one reason that children are more susceptible to lead poisoning. Suppose that Katrina absorbs 50% of her ingested lead. How many micrograms of lead will Katrina's blood system absorb each day from drinking ¾ of a liter of the school's water?

2. Blood lead half-life and elimination rate

a) Lead in the bloodstream is gradually removed by the kidneys, and is excreted in the urine. If no additional lead is ingested, then the lead in the blood will decrease in an *exponential* manner. Estimates for the amount of time for 50% of the lead to be passed out of an adult's bloodstream range from 28 to 36 days.[4] In children the amount of time can be even longer. Assume that for a 10-year-old girl the amount of time is about 50 days. With a half-life of 50 days, show that the daily decay multiplier is about $M = 0.986$. *Hint: 10 micrograms of lead will exponentially decrease to 5 micrograms of lead in 50 days—what is the value of M?*

b) With a half-life of 50 days, what percent of the lead in a child's bloodstream is removed each day? What percent is retained each day?

3. Creating the model

You have almost enough information to create an affine model of how lead in Katrina's bloodstream will change from day to day. You have the exponential decay multiplier for lead in the blood, and you know how much lead Katrina's blood is absorbing each school day. There are a few problems though. You do not know how much lead Katrina consumes on weekends and holidays when she is away from school. To overcome this difficulty, assume that the ¾ liter of school water she drinks daily is an *average* for all 7 days of the week.

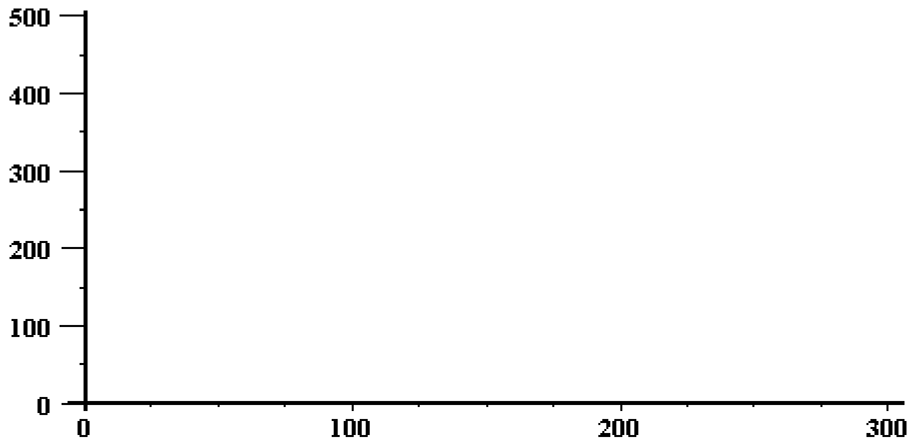
You also need a standard time to measure Katrina's blood lead level (BLL) each day, and you must decide the *order* in which the exponential decay and the linear absorption of lead take place (for writing your difference equation). Assume that Katrina's school day starts at 8am, and that is when her BLL is measured. Also assume that the exponential decay happens throughout the day, and Katrina drinks her water at the end of the day, right before 8am.

a) Let $u(n)$ represent the amount of lead in Katrina's bloodstream in micrograms on day *n*. Using the above assumptions, write the affine difference equation in terms of $u(n-1)$.

b) Recall that Katrina is just starting to attend this particular school, so assume that she has no lead in her bloodstream at the start of day $n=0$. Write the initial condition. *Include units!*

c) Enter the difference equation and initial condition into your technology device. Use a table to explore what happens to Katrina's blood lead level (BLL) over the 10-month (300-day) school year. In the space below describe what you find. *Hint: set your table to jump by increments of 50 days.*

d) Using technology, plot the difference equation over the 300-day period, to illustrate Katrina's projected BLL over a full school year. Set the graphing window dimensions equal to those of the graph below. Make a sketch below; include labels.



e) Determine the equilibrium value for the difference equation using algebra. *Add a horizontal dashed line to your graph to indicate the equilibrium level. Include units!*

4. A new classroom

Half-way through the school year, at the start of day $n = 150$, Katrina must move to a different classroom. She'll stay in that classroom for 2 days, then return to her original classroom at the start of day $n = 152$. The drinking fountain near this new classroom has extensive lead-based solder, polluting the water at a lead concentration of 200 ppb. After drinking ¾ of a liter of water from this fountain for 2 days, what will happen to Katrina's BLL? After she moves back to her original classroom, what will happen to her BLL? To answer these questions, forge on!

a) Begin by recording the amount of lead in Katrina's blood when she moves into the new classroom. Round to 2 decimal places.

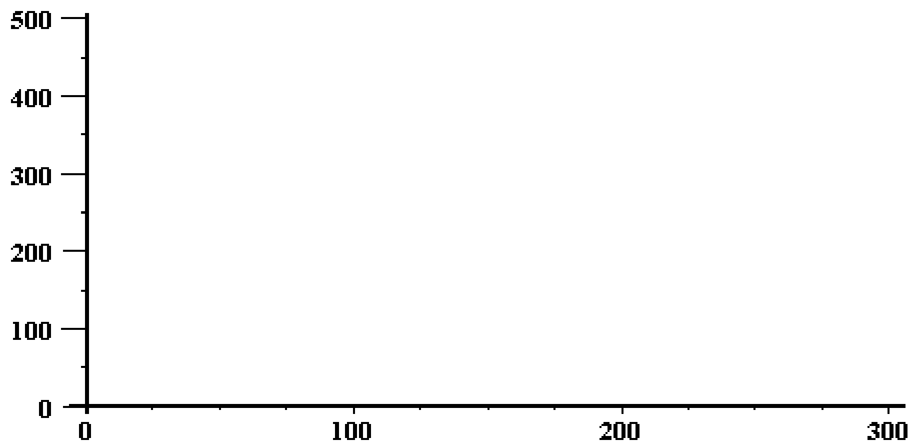
b) Now find the amount of lead that Katrina's blood will absorb daily from the new classroom's drinking water. Express your answer in micrograms. *You'll need to repeat several of the computations from question 1.*

c) Modify your old difference equation to create a new model for the lead dynamics in Katrina's blood while she is in the new classroom. What is the new difference equation?

d) Previously you determined Katrina's BLL when she entered the new classroom (day $n = 150$). Use that amount, and the new difference equation, to determine the amount of lead in Katrina's blood one day later (day $n = 151$). *Show the calculation.*

e) Now use the difference equation once again to find Katrina's BLL one day later (day $n = 152$). *Again show the calculation.*

f) At the start of day $n = 152$, Katrina moves back into her original classroom. Her BLL has been elevated substantially over the past 2 days. Re-draw the graph that you sketched previously, but stop after you reach day $n = 150$. On this new graph indicate the spike in Katrina's BLL on days $n = 151$ and $n = 152$. Also, indicate the previous equilibrium level with a horizontal dashed line. Include labels.



g) Now determine what happens to Katrina's BLL for days $n = 153$ to $n = 300$, after she has returned to her old classroom. Make use of the BLL on day $n = 152$ and the original difference equation. Add the results to the above graph. Describe below *how* you determined what would happen to her BLL.

h) What does the last graph that you sketched tell you about the *stability* of the original equilibrium level? In other words, is the equilibrium level stable or unstable? Explain.

5. Blood poisoning

You know that Katrina's BLL became very high during the school year. But was that level high enough to be dangerous? The Centers for Disease Control and Prevention (CDC) considers it dangerous if a child's blood lead concentration (BLC) reaches or exceeds 10 micrograms of lead per deciliter of blood ($\text{BLC} \geq 10\,\mu\text{g/dL}$). It is common to express hazard levels in terms of concentrations.

To determine if Katrina's BLC reached a dangerous level, you'll need an estimate of the amount of blood in a typical 10-year-old girl. Generally speaking, the bigger the person, the more blood he or she will have. One rule of thumb is that the blood volume for 10-year-old children is 75 milliliters (mL) for every 1 kilogram (kg) of body mass.[5] This means that the blood volume can be computed by the formula:

$$\text{Blood volume in milliliters} = 75\frac{\text{mL}}{\text{kg}} \times \text{body mass in kilograms}$$

a) The CDC publishes growth charts that display weight-for-age graphs (see attached sheet). Read the chart carefully to estimate Katrina's weight (mass) to the nearest whole kilogram. *Assume that she is at the 50th percentile by weight, meaning that half of the girls her age weigh less, and half weigh more.*

b) Now use the formula above to estimate the volume of blood in Katrina's bloodstream. Express answer in milliliters.

c) BLCs are typically computed using blood volumes that are expressed in deciliters. How many deciliters of blood does Katrina have? *Recall that there are 10 deciliters in 1 liter.*

d) Now that you know how many deciliters of blood are in Katrina's body, you can determine her BLC on any day of the school year. Start by computing Katrina's BLC on day $n = 150$, immediately before she moved to her temporary classroom. How much lead was in her blood on that day? What was the BLC in micrograms per deciliter?

e) Did Katrina's BLC reach a dangerous level before the 150th day of the school year? Explain.

f) The CDC would like to know how many days a typical 10-year-old girl (such as Katrina) can drink water with a lead concentration of 15 ppb before becoming lead poisoned (BLC = 10 μg/dL). You could work with your graph, but a more precise computation can be made using a solution equation. Compute the solution equation to Katrina's difference equation and initial condition. Then use the solution equation to determine the exact day when Katrina's BLC would reach 10 μg/dL . Show all work below.

g) You should have found that Katrina becomes lead poisoned well before the 150th day of the school year. But recall that she was drinking water with a lead concentration of 15 ppb—the maximum allowable concentration set by the EPA. It's possible that the EPA limit is too high, but it's also possible that something in your model is either wrong or missing. List one aspect of your model (an assumption, a constant, etc) that you think might be erroneous. Then modify your difference equation model and investigate how Katrina's BLL would change over time. Does your new model predict that Katrina will become lead poisoned? Explain.

References

[1] Deborah Bach, "Lead Woes Years in the Making," *Seattle Post Intelligencer*, August 10, 2004.

[2] U.S. Environmental Protection Agency PA, "Lead in Drinking Water," http://www.epa.gov/safewater/lead/index.html.

[3] Gabriel M. Filippelli et al., "Urban Lead Poisoning and Medical Geology: An Unfinished Story," *GSA Today*, January, 2005.

[4] Agency for Toxic Substances and Disease Registry, "Case Studies in Environmental Medicine: Lead Toxicity," http://www.atsdr.cdc.gov/HEC/CSEM/lead/

[5] Geigy Scientific Tables, 7th ed.:1971. As cited at the website: University of Michigan Medical School: Pediatric Research. "Guidelines: Children as Research Subjects," http://www.med.umich.edu/irbmed/InformationalDocuments/childguide.html.